Abstract:

This paper studies the empirical validity of a novel parameterization of a correlation matrix called the Generalized Fisher Z-Transformation. This parameterization has a number of desirable characteristics, including increasing Gaussian properties and ensuring a positive definite correlation matrix. The empirical validity is tested through including it as a signal in DCC-GARCH variance forecasting models and comparing its additive forecasting value to traditional DCC-GARCH models and DCC-GARCH models with realized measures of volatility as a signal. The accuracy of the forecasted conditional variances are tested using in-sample and out-of-sample estimated log-likelihood. Additionally, the forecasted variances are also compared through a global mean-variance portfolio optimization process, calculating weights from the forecasted conditional covariance. This research found that including the GFT as a signal improves on traditional DCC-GARCH models and performs similarly to DCC models augmented with realized volatility during in-sample evaluation. Similarly, in out-of-sample evaluation, the GFT models is dominant to the DCC-GARCH model. However, it performs significantly worse than the model augmented with realized volatility.

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I. Introduction and Motivation

Conditional variance and covariance forecasting of asset returns is a frequently tread path of research in financial economics. Increased ability of volatility forecasting can give serious insight to future portfolio risk, inform portfolio optimization, and a number of other financial applications. A key focus of this research includes estimating high-dimensional conditional covariance matrices and using high-frequency data for this estimation, both of which pose significant difficulties. This research adds to previous research surrounding variance-covariance forecasting by testing the empirical validity of a parameterization of a correlation matrix, defined as the Generalized Fisher Z-Transformation (GFT), and its ability to increase forecasting power of conditional covariance matrices. Specifically, the accuracy of the GFT forecasted conditional covariance matrices will be compared to the forecast accuracy of other, well tested, methods, including a traditional DCC-GARCH forecasting model and a DCC-GARCH model augmented to include realized correlation as a signal.

This research draws on a novel transformation of a correlation matrix proposed by Ilya Archakov and Peter R. Hansen. This transformation is another in a long line of parameterizations of covariance and correlation matrices that impose structures and restrictions on the matrices to ensure requisite characteristics of a covariance or correlation matrix, such as positive-definiteness and Gaussian properties. The GFT is a relatively unique parameterization, as it requires few restrictions on the covariance matrix, which allows for greater flexibility during modeling and in use. Additionally, if the GFT is used in concert with shrinkage, which brings extreme values closer towards a desired value, the GFT model may perform even better, as any error incurred in estimating the pre-transformed correlation matrix may be decreased.

This flexibility and structure allows for easy incorporation into already well-defined models with correlations as inputs. This research focuses on whether the GFT can add value to variance-covariance forecasting, testing both the possible improvement on in-sample and out-of-sample variance-covariance forecasting. To test the added value of the GFT, I will also compare forecasting power of the GFT to the forecasting power of realized measures of variance-covariance, which are a common improvement on traditional multivariate GARCH models. I will use a Dynamic Conditional Correlation Generalized Autoregressive Conditional Heteroscedasticity Model (DCC-GARCH) for forecasting, arguably the most commonly used variance-covariance forecasting model due to its parsimonious nature and flexibility with high-dimensional data. The GFT replaces one of the traditional regressors in the DCC-GARCH models, acting as a signal for future covariance matrices. Let this model be called the GFT-DCC model. The forecasting power of this model will be compared to a tradition DCC-GARCH model, which will be simply called the DCC model. A DCC-GARCH model with realized correlation acting as a signal, which will be called the RC-DCC model.

Outside of the log-likelihood criteria, a more theoretical in-sample and out-of-sample forecast evaluation, the forecasted covariance matrices will also be used in an out-of-sample portfolio optimization. Portfolio weights will be computed for each out-of-sample covariance matrix estimated, optimizing the weighting for each asset for minimum portfolio variance. This will serve as a more applied check of the GFT's empirical validity, leading to a more robust understanding of the GFT's power as a correlation parameterization, particularly in financial applications. Again, this will be compared to the portfolios determined by the conditional covariance matrices estimated through the traditional DCC and DCC with realized correlation inputs.

Theoretically, the GFT-DCC estimated conditional covariance matrices should have a better in-sample fit and have a more accurate out-of-sample forecast. This is for similar reasons that the RC-DCC model typically outperforms the DCC-GARCH: it has a more informative signal of the previous day's actual return and variance. The GFT-DCC should also, theoretically, produce portfolios with lower variance than the DCC. However, the RC-DCC has already been shown empirically that it outperforms the DCC-GARCH. Therefore, this research tests two questions: does the inclusion of the GFT improve on the DCC model and, if so, does it outperform previous improvements to the DCC model?

I found that both the GFT-DCC model and the RC-DCC outperform the DCC-GARCH model, as expected when considering in-sample criteria. The RC-DCC and GFT-DCC models perform almost identically, with RC-DCC barely outperforming the GFT-DCC model. Similarly, RC-DCC and GFT-DCC models are dominant to the DCC-GARCH model regarding the out-of-sample period. However, unlike the in-sample, the RC-DCC model significantly outperforms the GFT-DCC models. The out of sample evaluation may be subject to issues caused by the relatively volatile return environment of the forecast period when compared to the estimation period. There is plenty of room for further research regarding the validity of this model regarding forecasting power, however, as many alterations of DCC models have specific strengths and weaknesses when compared to other models, and this particular research may not have exploited the GFT to its strongest potential.

The data used is minute-by-minute price data from 20 of the assets in the Dow 30 between the beginning of 2016 through 2018, with 2016-2017 serving as the estimation period, where the model parameters will be estimated, and 2018 serving as the forecast period. Daily returns are taken as the log difference between the last price of the day and the first price of the day, and the highest frequency of returns used is 5 minutes.

II. Literature Review

It is a well-recognized phenomenon that the variance of asset returns with periods of little to no change in variance offset with occasional clustered moments of high volatility. The same fact holds for the covariance between two assets' returns. Therefore, it is important to study conditional variance and covariance, specifically how to estimate the conditional variance and covariance at specific times, as they are useful for many financial applications.

Arguably, the most prolific method of measuring conditional variance of asset returns is with a Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model, famously proposed by Robert Engle (1982). While this is not a model solely for modeling variances of asset returns, it is commonly used for said task. From this univariate method of modeling conditional variance at a specific time have grown a family of GARCH models, both univariate and multivariate, which estimate the conditional covariance between asset returns, and the DCC-GARCH is a member of this family. These models typically use daily returns, which are relatively low-frequency, to estimate conditional variances.

The DCC-GARCH model was first proposed by Robert Engle (1999) and is still commonly used to model covariance of between multiple variables. Following bulkier multivariate GARCH models such as MGARCH-BEKK and vech-GARCH, DCC-GARCH allows for much more parsimonious model (Engle & Kroner, 1995). Previous models required an inordinate number of parameters to estimate, greatly restricting the size of covariance matrix estimated. DCC-GARCH models model the conditional variance and covariance separately.

While less parsimonious models may give a better fit of the in-sample data as found by Huang et. al (2010), these models pick estimation error or extreme parameter value in forecasts, normally leading to better forecasts coming from DCC-GARCH models. Caporin and Mcaleer (2010) found that Scalar MGARCH-BEKK, which has a significantly lower number of parameters than traditional MGARCH-BEKK, and DCC-GARCH were found to have similar positive and negative characteristics However, DCC-GARCH is better fit to include alternate signals, which will be discussed in more depth in the models section. Therefore, DCC-GARCH is seemingly the best fit for this research.

Including higher-frequency returns in the estimation of variances and covariance matrices can result in better-fit models and more accurate forecasts. Including only low-frequency returns as signals in GARCH models may not give realistic estimates of conditional variance and covariance matrices. Particularly, it may take such GARCH models many days to catch up to some kind of shock, and using intraday, higher frequency return information may increase this catch-up time and overall fit of the model (Andersen et. al, 2000). This includes realized variance, which have been shown to fairly accurately measure daily conditional variance by Barndorff-Nielsen et. al (2002). This method can be extended to measures of realized covariance and realized correlation, which use intraday data to estimate conditional covariance and correlation for said day. This suggests that, assuming covariance is autocorrelative, realized measures can act as a useful signal for next-day conditional measures of volatility.

Regarding the two-step estimation, the selection of how to estimate the conditional variances of each asset individually requires justification. A univariate GARCH model requires the assumption that volatility react symmetrically to positive and negative shocks, and the coefficients on the ARCH and GARCH terms must be positive (Engle, 1982). The exponential GARCH model (EGARCH), proposed by Nelson (1991), allows these assumptions to be relaxed. This is a log-linear model that includes a “leverage function” that allows for asymmetrical reactions. A possible drawback of this model, however, is that it is not stationary; therefore, it may result in an extreme and unrealistic estimated unconditional variance (Kim & Lee, 2006). While this is a valid concern, the benefits of asymmetrical effects and unbounded coefficients outweighs the concerns over stationarity.

Additionally, EGARCH is the chosen model because it has a variation, known as the Real-EGARCH, which allows for the incorporation of realized volatility measures in its estimation. This model was proposed by Hansen et. al (2010), which showed that Real-EGARCH outperformed the traditional univariate GARCH and EGARCH models in in-sample fit and variance forecasting. Conditional variances will be estimated with both an EGARCH model and a Real-EGARH model and the performance of the conditional variances will be compared in performance and the better will be used to estimate the forecasted conditional covariance matrices.

Now, let us switch focus to developments made in covariance parameterization and the motivation for the GFT. According to research conducted by Fan et al (2008), a structure is needed to decrease the estimation error of a covariance matrix. This structure can take a number of forms, including macroeconomic factor models, as presented by Beenstock and Chen (1986) or a kernel based estimator. Since the model uses daily returns, most macroeconomic factors are not released at a high enough frequency to well-fit the data. A kernel based estimator requires assumptions which, if don’t accurately describe the data, can result in a misspecified density estimation. Without significant prior research on this specific dataset, the kernel based estimator would likely be misestimated or, at least, would not be the best fit for the model. The GFT falls into neither of the above categories, which is beneficial because it requires fewer assumptions than the above models and is well fit for most financial data.

There have been proposed log-transformations of covariance matrices used to decrease estimation error in the past. Leonard and Hsu (1992) proposed a log-transformation based off of a Volterra integral equation. However, this uses Bayesian techniques and requires the covariance matrix to be positive-definite. The GFT relaxes such assumptions.

The Fisher Z-Transformation is a method of transforming a Pearson correlation coefficient, which is bounded by [-1 1], into a continuous and unbounded value. Since the correlation coefficient is bounded, the sample distribution of this value for correlated variables is skewed. The Fisher Z-Transformation decreases the skewness of the correlations by extending the range from [-1 1] to ($-\infty$ ∞), which thereby increases the Gaussian properties of the correlation measures (Corey et. al, 1998). There is also a well-known Fisher Z-Transformation for a (2 x 2) correlation matrix, which follows directly from the previously described method. Again, this transformation increases the Gaussian properties of the (2 x 2) matrix; however, this transformation is limited for a (2 x 2) correlation matrix. The GFT is a proposed generalized Fisher Z-Transformation for a correlation matrix of any dimension. Therefore, if the GFT is applied to a correlation matrix that is not normally distributed, the transformed matrix will have increased Gaussian properties. This will allow for shrinkage estimators to be applied.

In 1961, James and Stein proposed their currently well-known shrinkage estimator, which they showed to improve the risk measurement when compared to the sample covariance matrix.

This method essentially pulls in all of the estimates closer to the mean, decreasing the outliers that are a result of the covariance estimation and parameterization. However, James-Stein shrinkage requires the shrunk variables be relatively normally distributed to effectively shrink the covariance matrix (Daniels & Kass, 2001). Therefore, since GFT increases the normality of the estimations, the transformed matrix will be well suited for James-Stein shrinkage.

III. Data Review

The data used is one minute tick data for all of the equities that are components of the Dow 30 for every trading day from 01/04/2016 – 11/08/2018, with the estimation period from 01/04/2016 – 12/29/2017 and the training period from 01/02/2018 – 11/07/2018. There are 713 trading days covered in the dataset, with 501 days in the training period and 211 days in the estimation period.[1] The data includes date, timestamp, ticker, open price, high price, low price, close price, total volume, total quantity, and- total trade count. The only data that is going to be used is date, timestamp, ticker, open price and close price. The price of the asset for each minute will be determined as the average of the open and close price.

There are 20 equities listed below in table II.1. There has been one change to the composition of the Dow 30 since the beginning of 2016; Dow Chemical merged with DuPont, creating the new company DowDuPont with ticker ‘DWDP’, which was the original ticker of DuPont. In order to remove the change in price that is inherent in merging a company, tickers ‘DWDP’ and ‘DD’ were removed from the dataset, where ‘DD’ was the ticker for Dow Chemical.

[1] Due to restrictions of the data used, there are a small number of days during the training period that are removed from the dataset. These are relatively few in number, so this shouldn’t have a significant impact on the model.

Table II.1: Companies in Dataset

Ticker	**Company Name**
CAT	Caterpillar Inc.
CVX	Chevron Corp.
DIS	Walt Disney Co.
GE	General Electric Co.
GS	Goldman Sachs Group Inc.
HD	Home Depot Inc.
IBM	International Business Machines Corp.
JNJ	Johnson & Johnson
JPM	JPMorgan Chase & Co
KO	Coca-Cola Co.
MCD	McDonald's Corp
MRK	Merck & Co. Inc.
PFE	Pfizer Inc.
PG	Procter & Gamble Co.
UNH	UnitedHealth Group Inc.
UTX	United Technologies Corp.
V	Visa Inc.
VX	United States Steel Corporation
WMT	Walmart Inc.
XOM	Exxon Mobil

The rest of the assets that are members of the Dow 30 that were not used in this research were removed due to the paucity of data that the Quandl database had on these assets for each day. The database might have data for only certain times during certain days or might not include data for every asset for every trading day, both of which shrunk pool of useful data significantly.

Table II.2: Correlation of Daily Returns in Estimation Period

-	CAT	CVX	DIS	GE	GS	HD	IBM	JNJ	JPM
CAT	1.00								
CVX	0.50	1.00							
DIS	0.32	0.34	1.00						
GE	0.39	0.35	0.28	1.00					
GS	0.42	0.35	0.29	0.34	1.00				
HD	0.27	0.28	0.31	0.25	0.27	1.00			
IBM	0.39	0.40	0.31	0.28	0.31	0.31	1.00		
JNJ	0.21	0.22	0.22	0.19	0.20	0.18	0.21	1.00	
JPM	0.46	0.46	0.40	0.46	0.72	0.28	0.33	0.27	1.00
KO	0.14	0.18	0.19	0.23	0.04	0.29	0.28	0.31	0.06
MCD	0.17	0.18	0.17	0.17	0.17	0.29	0.22	0.27	0.21
MRK	0.22	0.26	0.25	0.25	0.25	0.21	0.36	0.52	0.31
PFE	0.21	0.24	0.24	0.22	0.22	0.26	0.27	0.51	0.31
PG	0.20	0.24	0.21	0.19	0.05	0.15	0.31	0.28	0.06
UNH	0.25	0.17	0.24	0.15	0.26	0.18	0.20	0.34	0.27
UTX	0.44	0.38	0.26	0.36	0.34	0.24	0.43	0.29	0.39
V	0.37	0.32	0.25	0.28	0.40	0.37	0.42	0.28	0.37
WMT	0.12	0.19	0.23	0.18	0.05	0.22	0.21	0.21	0.17
XOM	0.45	0.70	0.30	0.38	0.29	0.21	0.35	0.28	0.45
VX	0.20	0.30	0.27	0.27	0.11	0.20	0.32	0.27	0.20

	KO	MCD	MRK	PFE	PG	UNH	UTX	V	WMT	XOM
KO	1.00									
MCD	0.27	1.00								
MRK	0.20	0.20	1.00							
PFE	0.22	0.23	0.56	1.00						
PG	0.49	0.24	0.22	0.18	1.00					
UNH	0.26	0.27	0.35	0.35	0.18	1.00				
UTX	0.25	0.26	0.32	0.32	0.20	0.24	1.00			
V	0.31	0.30	0.28	0.30	0.23	0.25	0.40	1.00		
WMT	0.24	0.21	0.25	0.21	0.28	0.19	0.19	0.19	1.00	
XOM	0.21	0.18	0.28	0.23	0.26	0.15	0.31	0.22	0.22	1.00
VX	0.35	0.16	0.25	0.18	0.28	0.14	0.20	0.17	0.21	0.29

All of the assets are positively correlated over this period, with the average correlation 0.27. It is logical that all of the assets are positively correlated as they are all large-cap, US based companies.[2]

A point of note is the difference in the trend of returns during the estimation period and the forecast period. During the estimation period, the overall market was performed extremely well, with the Dow 30 growing 13.42% and 25.08% in 2016 and 2017 respectively; however, over the forecast period, the Dow 30 fell 5.63%. This discrepancy might lead to forecasted conditional variances and covariance matrices that don't fit the return data in the forecast period well even though the estimated parameters fit the in-sample data well.

In order to maximize the frequency of data used without inducing a concerning amount of microstructure noise into the data, five minute data will be used to calculate intraday returns. However, in order to use all of the data possible, five minute returns will be taken five times every five minutes and then averaged for each day, meaning five minute ticks will be taken at minutes 1,6,11,16,…, and then another group taken at minutes 2,7,12,17,…, continuing on until the fifth group is taken at minutes 5,10,15,20,…, etc. The five estimates of returns are then averaged together. This should smooth out any random extreme price discrepancies. This is simply a method to use all of the data provided at 1-minute intervals without introducing the microstructure noise that is inherent in 1-minute data.

IV. Method

[2] The correlation of daily asset returns during the forecast period is listed in the appendix (II.3)

This section has the goal of outlining the theoretical backing and empirical models used in this research. First, the following is the basic notation used throughout the rest of the paper:

Let $p_{i,t,n}$ be the n^{th} price of asset *i* on day *t*, and let $r_{i,t,n}$ be the log 5-minute return at on day *t* at time *n*.

$$r_{i,t,n} = \ln(p_{i,t,n}) - \ln(p_{i,t,n-5}) \quad (4.1)$$

Let R_t be a matrix (*i* x *n*) of the returns for time *t* with *n* returns for all *i* assets.

$$R_t = \begin{bmatrix} r_{1,t,1} & \cdots & r_{1,t,n} \\ \vdots & \ddots & \vdots \\ r_{i,t,1} & \cdots & r_{i,t,n} \end{bmatrix} \quad (4.2)$$

Let RCv_t be the realized covariance matrix on day *t*, and let RC_t be a realized correlation matrix on day *t*, with K_t as a matrix with the realized variances, which is the sum of squared intraday returns, on the diagonal and zeros on all of the off-diagonal elements. Additionally, let a superscript *T* of a matrix represent the transpose of said matrix.

$$RCv_t = R_{i,t} * R_{i,t}{}^T \quad (4.3)$$

$$RC_t = K_t^{-1} * RCv_t * K_t^{-1} \quad (4.4)$$

Let $y_{i,t}$ be the demeaned daily return of asset *i* of day *t*, let $\overline{y_i}$ be the average daily return of asset *i*, and let y_t be a vector of demeaned daily returns at time *t* for all assets *i*.

$$y_{i,t} = \ln(p_{i,t,close}) - \ln(p_{i,t,open}) - \overline{y_i} \quad (4.5)$$

$$y_t = \begin{bmatrix} r_{i,t} \\ \vdots \\ r_{20,t} \end{bmatrix} \quad (4.6)$$

i. Exponential GARCH

The first step in forecasting conditional covariance with a DCC-GARCH model is estimating conditional variances for each asset individually, which can be accomplished through a univariate GARCH model or one of the many variations on a univariate GARCH. In this research, I will be estimating the conditional variances for each asset through an EGARCH model. This model has many advantages over a traditional GARCH model, as discussed in the literature review, but it also does not guarantee the model is stationary, so there may be no estimated unconditional variance. Therefore, in addition to an EGARCH model, I will also estimate the conditional variances with an EGARCH model that includes measures of realized variance as a signal. The in-sample fit and out of sample accuracy will be measured and the model that is shown to better estimate the conditional variance will be used to forecast the DCC models.

Consider the following model for demeaned returns. Let $h_{i,t}$ be the conditional variance for asset *i* on day *t*, and let $z_{i,t}$ be defined as returns which are independent and identically distributed (i.i.d.) with an expected value of 0 and a variance of 1.

$$y_{i,t} = \sqrt{h_{i,t}} * z_{i,t} \quad (4.7)$$

Now consider the following specification of the EGARCH model:

$$\log(h_{i,t}) = \omega + \beta \log\left(h_{i,t-1}\right) + \tau_1 z_{i,t-1} + \tau_2\left({z_{i,t-1}}^2 - 1\right) \quad (4.8)$$

These parameters are estimated by maximizing a quasi-log-likelihood function. While none of the elements that make up (4.8) are directly observable, given a starting value $h_{i,1}$, $z_{i,t}$ can be estimated recursively through (4.7)

Also, consider the Real-EGARCH model as specified by Hansen et. al, which is made up of a return equation similar to the traditional EGARCH model and a measurement equation that estimates a realized volatility measure. Let $u_{i,t}$ represent the conditional variance signal derived from the realized volatility measure, and let $x_{i,t}$ be the actual estimated realized volatility measure:

$$\log(h_{i,t}) = \omega + \beta \log(h_{i,t-1}) + \tau_1 z_{i,t-1} + \tau_2({z_{i,t-1}}^2 - 1) + \theta u_{i,t-1} \quad (4.9)$$

$$\log(x_{i,t}) = \varepsilon + \varphi \log(h_{i,t}) + \delta_1 z_{i,t} + \delta_2({z_{i,t}}^2 - 1) + u_{i,t} \quad (4.10)$$

Similar to above, these parameters will be estimated by maximizing a quasi-log-likelihood as specified by Hansen et. al (Hansen et. al, 2012). It is important to note that, directly, the log-likelihood estimates of the EGARCH and Real-EGARCH models cannot be directly compared, as the Real-EGARCH includes the measurement equation, which results in a differently structured quasi-log-likelihood function to maximize. However, the Real-EGARCH assumes independence between $u_{i,t}$ and $x_{i,t}$, which allows for the quasi log-likelihood function to be split up. One of the partial log-likelihood functions is equivalent to the log-likelihood function of the EGARCH model. These are the log-likelihoods that are compared.

Both of these estimations will two different estimates of the conditional variance for each asset which will be used to studentize returns, as specified in (4.7). These models will be compared using the value of the maximized log-likelihood value of each asset for both models.

ii. Constant Conditional Correlation

The first of the estimated models is a Constant Conditional Correlation model, which was the predecessor to the DCC-GARCH model. Unlike the DCC model, the conditional correlation is assumed to be constant over the estimation and forecast period. Although this may not be a realistic assumption, it is important to include this measure in the comparison. Let $\bar{C}$ be defined as follows:

$$\bar{C} = \frac{1}{T}\sum_{t=1}^{T} z_{t-1} * z_{t-1}{}^{T} \quad (4.11)$$

Since this model requires no parameter estimation, the constant conditional correlation can be placed directly into log-likelihood measure that will be used to measure forecast accuracy along with the other estimated conditional covariance matrices from the DCC-GARCH models. It is important to note that CCC is nested inside of DCC-GARCH; therefore, DCC-GARCH should necessarily result in a higher in-sample log-likelihood value than the CCC model.

iii. DCC-GARCH

The goal of the DCC-GARCH model is to model the conditional covariance of asset returns by estimating the variances and covariances separately. After modeling the conditional variances through a univariate GARCH process, the covariances are estimated through a parameterized correlation matrix that ensures the modeled covariance matrix is positive definite. The parametrized correlation is modeled as a combination of the unconditional correlation between asset returns, the previous periods' conditional parameterized correlation, and some signal of previous periods' returns. This research will alter the DCC model by changing the last input, the signal of the previous periods' returns, seeing which signal can give the best information so that the model can best represent the true data generating process.

Similar to EGARCH model, the first step is to model the studentized returns. Let H_t be the conditional covariance matrix at day *t,* and let H_t be the conditional covariance matrix at time *t.*

$$y_t = {H_t}^{1/2} * z_t \quad (4.12)$$

This is a similar characterization as above in the EGARCH model, but it is in matrix form, considering returns of all assets in one model. Again, z_t is i.i.d., the expected value is 0, and $\mathrm{E}[z_t {z_t}^T] = I$. The conditional variances were estimated in the EGARCH models, and the conditional covariance will now be estimated by estimating a parameterization of the correlation matrix, where C_t is the correlation matrix at time *t.* Let D_t be a matrix of the conditional standard deviations, with the conditional standard deviations on the diagonal and zeros on the off-diagonals

$$H_t = D_t * C_t * D_t \quad (4.13)$$

In order to obtain a positive definite correlation matrix during estimation of parameters and throughout the forecasting process, let C_t be parameterized in the following way:

$$C_t = {F_t}^{-1} * Q_t * {F_t}^{*-1} \quad (4.14)$$

Let the parameters *a* and *b* be an estimated scalar where *a* and *b* are both positive and *a* + *b* < 1. $z_{t-1} * {z_{t-1}}^T$ acts as a low-frequency signal for future conditional variances.

$$Q_t = (1 - a - b)\bar{Q} + a(z_{t-1} * {z_{t-1}}^T) + b * Q_{t-1} \quad (4.15)$$

$$\bar{Q} = \frac{1}{T}\sum_{t=1}^{T} z_{t-1} * {z_{t-1}}^T \quad (4.16)$$

Additionally, let ${F_t}^*$ be specified as below. Let $q_{i,j}$ be the element of Q_t in row *i* and in column *j*.

$$F_t{}^{*} = \begin{bmatrix} \sqrt{q_{1,1}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{q_{20,20}} \end{bmatrix} \quad (4.17)$$

This type of parameterization forces that C_t is positive definite, where the diagonals are identically 1 and each element of the matrix must be bounded by [-1,1]. This model will simply be called DCC-GARCH.

The following is a model is an alteration of the DCC-GARCH model, replacing the previous signal with realized correlation, which uses intraday data as specified in (4.4). This model will be called RC-DCC:

$$Q_t{}^{RC} = (1 - a - b)\bar{Q} + aRC_{t-1} + bQ_{t-1}{}^{RC} \quad (4.18)$$

Both the DCC-GARCH and the RC-DCC models' parameters are estimated by maximizing the following function for T days in the estimation period:

$$\mathcal{L} = -\frac{1}{2}\sum_{t=1}^{T} \log(det(C_t)) + z_{t-1}{}^{T} * C_t{}^{-1} * z_{t-1} \quad (4.19)$$

This likelihood function assumes that asset returns are best described by the normal distribution.

iv. Generalized Fisher Z-Transformation

The GFT is a log-transformation of a sample correlation matrix that increases the normality of the correlation estimates. This paper gives a brief overview of Archakov and Hansen's proposed GFT, but the full explanation and derivation can be reviewed in their paper. The Z-Transformation is a method to transform Pearson's correlation coefficient so that it is normally distributed. Let z be the transformed correlation coefficient, and let ρ be Pearson's correlation coefficient. The formula follows:

$$z = \frac{1}{2}\log\left(\frac{1+\rho}{1-\rho}\right) \quad (4.20)$$

$$G = \begin{bmatrix} 1 & \frac{1}{2}\ln\left(\frac{1+\rho_{2,1}}{1-\rho_{2,1}}\right) \\ \frac{1}{2}\ln\left(\frac{1+\rho_{1,2}}{1-\rho_{1,2}}\right) & 1 \end{bmatrix} \quad (4.21)$$

The above is the natural Fisher Z-transformation of a correlation matrix between two assets. There is research surrounding an element-wise Fisher Z-Transformation for an n x n matrix, meaning each off-diagonal element is transformed according to (4.20). However, this approach does not fulfill desirable requirements for a parameterization that the GFT fulfills (Archakov and Hansen 2018).

If A is a 2x2 matrix, then if G = logm(A):

$$G = \begin{bmatrix} \frac{1}{2}\ln(1-\rho^2) & \frac{1}{2}\ln\left(\frac{1+\rho}{1-\rho}\right) \\ \frac{1}{2}\ln\left(\frac{1+\rho}{1-\rho}\right) & \frac{1}{2}\ln(1-\rho^2) \end{bmatrix} \quad (4.22)$$

It follows that the off-diagonal elements are individually transformed identically to (4.21). To clarify, logm(A) is the logarithm of a matrix, which is not the same as taking the element-by-element log of a matrix. Archakov and Hansen's paper argues that a generalized Fisher Z-transformation is given by the logarithm of a correlation matrix of any dimension.

To give a general definition of the GFT: let B be an (n x m) matrix of demeaned returns, with m individual assets and n observations. Then let H_t be the sample covariance matrix of $X_{i,t}$, as defined previously.

Then, transform the covariance matrix into a correlation matrix C_t at time *t*, such that:

$$C_t = \begin{bmatrix} 1 & \cdots & \blacksquare \\ \vdots & \ddots & \vdots \\ \frac{\sigma_{m,1}}{\sigma_m^2 \sigma_1^2} & \cdots & 1 \end{bmatrix} = \begin{bmatrix} 1 & \cdots & \blacksquare \\ \vdots & \ddots & \vdots \\ \rho_{m,1} & \cdots & 1 \end{bmatrix} \text{(4.23)}$$

The transformation of the correlation matrix is centered on the log transformation of the correlation matrix. The finalized GFT = γ(C) = offdiag(log(C)), where offdiag is a function that stacks the off diagonal elements in the lower triangle of the matrix log(C) into a vector, and log(C) is the log function applied to the matrix C. If C is a symmetric positive definite matrix, which is assumed for all of our correlation matrices, with the below eigendecomposition. Let Q_t be an orthonormal matrix such that $Q_t Q_t^T = I$, and Λ_t is a matrix with eigenvalues of C_t on the diagonal and zeroes on all of the off-diagonal elements.

$$\mathrm{C}_t = Q_t \Lambda_t Q_t^T \text{ (4.24)}$$

$\mathrm{logm}(\mathrm{C}_t)$, then takes the following form. Let $\log(\Lambda_t)$ be a matrix with the log of each eigenvalue of C_t is on the diagonal and the off-diagonal elements are all zero.

$$\mathrm{logm}(\mathrm{C}_t) = Q_t \log(\Lambda_t) Q_t^T \text{ (4.25)}$$

The finalized γ(C_t) is offdiag(logm(C_t)), which is a vector of the off-diagonals of log(C_t).

To clarify the offdiag() function, below is an example:

$$\mathrm{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \text{(4.26)}$$

$$\mathrm{offdiag(R)} = \begin{bmatrix} r_{21} \\ r_{31} \\ r_{32} \end{bmatrix} \text{(4.27)}$$

Let $\mathrm{offdiag(R)}$ be a vector of the stacked off-diagonal elements of R.

In their paper, Archakov and Hansen show that there is an isomorphism between the set of n x n non-singular correlation matrices and $\mathbb{R}^{\frac{n(n-1)}{2}}$ and they also provide an iterative program for the inverse mapping.

v. James Stein Shrinkage

James Stein shrinkage can be applied to the GFT transformed vector of off-diagonal correlation elements $\gamma(C_t)$ at the end of day *t*. The goal of this shrinkage is to decrease the magnitude of extreme estimates that are likely due to estimation error and parameter miss-estimation. The shrinkage of $\gamma(C_t)$ will take the following form. Let $\bar{\gamma}$ be a vector of the average transformed correlation of each element over the estimation period, let $\gamma(C_t)^{\,S}$ indicate the shrunk $\gamma(C_t)$ at time *t*, and let *s* be a vector of shrinkage intensities indicates the shrinkage intensity.

$$\gamma(C_t)^{\,S} = \bar{\gamma} + s(\gamma(C_t) - \bar{\gamma}) \text{ (4.28)}$$

This means the transformation correlation estimates will be shrunk towards the mean correlation of all of the assets at time *t*. It is important to note that the reasoning for $\bar{\gamma}_t$ being the shrinkage target. It is likely that the majority of these assets are positively correlated, so the accurate correlation of two assets is likely above zero, so if the shrinkage target is at zero, transformed correlation estimates could be shrunk away from the true value. Now, consider *s*, the shrinkage intensity, which will be calculated as follows. Let *i* be the number of assets, and let σ_i^2 be the variance of the correlation of each pair of assets over the estimation period. The smaller the s_i, the further the estimate is shrunk closer to the shrinkage target.

$$s_i = 1 - \left[\frac{(i-3)}{\sum_j^i(\gamma(C_t)_i - \bar{\gamma})^2}\sigma_i^2\right] \text{ (4.29)}$$

$$s = \begin{bmatrix} s_1 \\ \vdots \\ s_n \end{bmatrix} \quad (4.30)$$

Additionally, consider a double-shrunk estimator, which is simply altering (4.27) in the following way:

$$\gamma(C_t)^{S\_2} = \bar{\gamma} + \frac{S}{2}[\gamma(C_t) - \bar{\gamma}] \quad (4.31)$$

This model simply shrinks the transformed correlation matrix farther towards the desired shrinkage target. This could be desired if the realized correlation matrix is expected to show extreme values. For example, if returns are extremely volatile, applying double shrinkage can possibly increase the explanatory power of the transformed realized correlation as a signal, removing the extreme values that could throw the estimation off and result in a poor fit and inaccurate forecast.

vi. GFT-DCC

The following specification is essentially the same model as the RC-DCC with signal of shrunk realized correlation.

The following is the specification of the GFT-DCC model. Let $\bar{\gamma}$ be defined as above and let $\gamma^{-1}(x)$ be the inverse mapping from $\mathbb{R}^{\frac{n(n-1)}{2}}$ to the space of (n x n) non-singular correlation matrices. For clarification, K_t is the signal that will be constructed by applying the GFT to a realized correlation matrix, applying shrinkage to the transformed correlation matrix, and then mapping the shrunk transformed matrix back into $\mathbb{R}^{\frac{n(n-1)}{2}}$ space.

$$Q_t^{GFT} = (1 - a - b)\bar{Q} + aK_{t-1} + bQ_{t-1}^{GFT} \quad (4.32)$$

$$K_t = \gamma^{-1}(\gamma(\mathrm{C}_t)^{S}) \quad (4.33)$$

These parameters will be estimated with the same log-likelihood function as above.

Lastly, consider the previous model with a double-shrunk transformed realized correlation matrices as defined in (4.30) with name GFT-DCC-2:

$$Q_t^{GFT} = (1 - a - b)\bar{Q} + a\Psi_{t-1} + bQ_{t-1}^{GFT} \quad (4.34)$$

$$\Psi_t = \gamma^{-1}(\gamma(\mathrm{C}_t)^{S\_2}) \quad (4.35)$$

This model is included because the shrinkage intensities calculated for this dataset, as defined in (4.28), are relatively high, meaning any extreme values that are desired to be shrunk towards the shrinkage target are not moved far out of extremity. The average shrinkage intensity is 0.8369, which may not shrink the data enough to avoid extreme values that taint the estimation. The comparison of the performance of a model with

Similar to above, these parameters will be estimated by maximizing (4.19).

vii. Log-Likelihood

The goodness of fit of the in-sample estimations and the out-of-sample forecasts of the models and will be compared by using log-likelihood estimates. Assuming these models are estimated with the same likelihood function, or, in the case of EGARCH and Real-EGARCH, can be reduced to the same likelihood function, the output of the log-likelihood function given the maximized parameters can show how well a model fits its data. This is crucial, because the true conditional variance and covariance are not observable, so there is nothing to compare the estimated and forecasted variances against. This will allow a robust comparison of the two

EGARCH models and a comparison between all of the different DCC models when the true conditional variance and covariance matrices are unknown.

The likelihoods of each model will be compared through some criterion such as the Akaike information criterion (AIC), which gives an estimate of the relative quality of statistical models for each set of data given the likelihood of each function and the number of parameters estimated. The model with the lowest AIC value is suggested to be the best fit model. The AIC values from each model can then be compared to interpret the probability that a model minimizes the relative information loss from the true data generating process when compared to the supposed dominant model.

Let AIC_i be the estimated AIC value of asset *i*, let *p* be the number of parameters estimated, and let $\mathcal{L}_i$ be the maximized log-likelihood for model *j*.

$$\text{AIC}_i = 2 * p - 2 * \mathcal{L}_i \quad (4.36)$$

Then the AIC values can be compared to find the relative goodness of fits of two models as specified below. Let *i_low* represent the model that has the minimum AIC_i, let ω_i be the relative probability that model *i* minimizes information loss when compared to model *i_low*.

$$\omega_i = \exp\left(\frac{\mathcal{L}_{i\_low} - \mathcal{L}_i}{2}\right) \quad (4.37)$$

For example, if $\omega_{DCC-GARCH}$ = .005 and it is being compared to GFT-DCC, then DCC-GARCH is .005 times more likely to minimize information loss of the true data generating model than GFT-DCC.

It is important to note that all of the models require the estimation of two parameters, therefore if the log-likelihood of a given model is higher than the log-likelihood of another model, then the AIC is necessarily going to suggest that the model with the higher log-likelihood is the dominant model. Therefore, it is not necessary to compute the AIC values to find the model with best fit and forecasting value. However, computing the AIC values allows for a direct way to compare the power of each model through (4.37).

viii. Mean-Variance Optimization

A second method to test the accuracy of the forecast is to optimize a portfolio based off of the forecasted conditional variances based on some certain criteria. For this research, the obvious choice is global mean-variance portfolio optimization, which minimizes portfolio return variance, independent of asset returns. This method allows for a truly out-of-sample test of the forecasted covariance without having to also include forecasted returns in the estimation of weights, which would not be desirable as then the performance of the portfolio would depend jointly on the forecasted returns and the forecasted variance.

Consider the following specifications of the optimization model. Let r_t^p be the return of the portfolio at time *t*, let $p_{i,t}$ be the open-to-close return of asset *i* at time *t*, let P_t simply be a 20x1 vector of stacked daily returns for each asset, and let w_t be a 1x20 vector of portfolio weights at time *t*.

$$p_{i,t} = \ln\left(p_{i,t,close}\right) - \ln\left(p_{i,t,open}\right) \quad (4.38)$$

$$P_t = \begin{bmatrix} p_{1,t} \\ \vdots \\ p_{20,t} \end{bmatrix} \quad (4.39)$$

$$r_t^p = w_t * P_t \quad (4.40)$$

The portfolio weights is what are being optimized to minimize portfolio variance, which is defined as follows. Let σ_P^2 be the portfolio variance and let H_t be the conditional covariance matrix at time *t* as defined previously.

$$\sigma_P^2 = w_t * H_t * w_t \quad (4.41)$$

Certain restrictions are placed on w_t, specifically that the weights must sum to one, and each must weight is bounded by [-1,1].

A weight vector will be estimated for each day for each of the six DCC models, and whichever model estimates weights that allow for the lowest variance in portfolio returns over the forecast period is considered the model with the best forecast. The higher the variance of portfolio returns over the forecast period, the less accurate the forecasted variance.

V. Results

i. EGARCH and Real-EGARCH Models

In this section, I present the results of the EGARCH and Real-EARCH models, discuss the estimated coefficients, and compare the in-sample and out-of-sample log-likelihoods for each asset.

Referencing Table V.1, it is important to note that almost all of these coefficients are significant, with the insignificant coefficients largely being made up by the intercept estimates ω and ε. Since the goal of this research is not to evaluate the difference between EGARCH and Real-EGARCH, I will not go into a significantly in-depth analysis of each coefficient; however, an analysis of **θ** is important as it measures the impact of realized variance on the model of conditional variance.

Table V.1: Coefficients of Real-EGARCH Model[3]

	ω	β	τ1	τ2	θ	ε	φ	δ_1	δ_2
CAT	-0.935	0.916	0.077	0.130	0.353	-4.072	0.595	0.008	0.101
CVX	-0.743	0.934	-0.078	0.061	0.491	-2.671	0.706	-0.010	0.048
DIS	-3.427	0.704	0.000	0.097	0.645	-5.121	0.492	-0.013	0.055
GE	-4.552	0.607	0.013	0.153	0.710	-6.751	0.335	0.032	0.057
GS	-8.115	0.277	0.000	0.261	0.312	-8.713	0.117	0.001	0.052
HD	-3.328	0.713	-0.068	0.130	0.513	-5.137	0.487	-0.024	0.077
IBM	-1.390	0.884	0.055	0.109	0.458	-4.990	0.509	0.032	0.068
JNJ	-9.102	0.258	-0.030	0.234	0.352	-8.240	0.238	0.004	0.065
JPM	-3.152	0.716	0.031	0.103	0.370	-3.500	0.629	0.023	0.052
KO	-3.177	0.743	-0.077	0.160	0.638	-6.577	0.382	-0.016	0.055
MCD	-7.395	0.402	0.002	0.220	0.385	-9.417	0.124	-0.017	0.023
MRK	-3.373	0.710	-0.002	0.152	0.604	-5.610	0.440	0.006	0.061
PFE	-2.384	0.792	0.016	0.134	0.542	-4.174	0.566	0.014	0.092
PG	-3.458	0.715	0.086	0.145	0.591	-5.825	0.443	0.041	0.074
UNH	-2.229	0.797	0.025	0.062	0.578	-4.702	0.531	0.005	0.068
UTX	-2.967	0.743	0.080	0.094	0.586	-4.782	0.534	0.054	0.065
V	-1.869	0.843	0.037	0.146	0.515	-4.468	0.553	0.003	0.082
WMT	-6.630	0.438	0.047	0.200	0.298	-8.641	0.178	0.037	0.053
XOM	-1.222	0.898	-0.052	0.128	0.567	-3.529	0.633	0.008	0.075
VX	-9.715	0.163	-0.015	0.235	0.201	-8.782	0.161	0.047	0.059

All of the coefficients are positive and significant, suggesting that there is a substantial relationship between lagged realized variance and conditional variance. Additionally, between EGARCH[4] and Real-EGARCH, the β coefficient's magnitude dropped by a non-insignificant amount without a large change in the standard errors, suggesting that the β coefficient loses some explanatory power when including a measure of realized variance.

[3] A table of robust standard errors are available in the appendix (V.2)
[4] A table of coefficients of the EGARCH model and a table of robust standard errors of these coefficients are available in the appendix (V.2, V.3 respectively)

Table V.5: Average Log-Likelihood of EGARCH and Real-EGARCH

	In-Sample	**Out-of-Sample**
EGARCH	1771.36	-3095.91
Real-EGARCH	1883.32	664.63

Table V.5 gives further credence to Real-EGARCH outperforming EGARCH both in-sample and out-of-sample.[5] During the estimation period, Real-EGARCH outperforms EGARCH for every asset. During the forecast period, there are assets where Real-EGARCH outperforms EGARCH and visa-versa. The discrepancy in the average out-of-sample log-likelihood can be blamed on a small number of assets displaying extremely negative log-likelihood values. This is likely due to the EGARCH model's inability to ensure the existence of an unconditional variance. As the EGARCH process moves further into the forecast period, the forecasted conditional variance converges to the unconditional variance; however, if the unconditional variance does not exist, the forecasted variances may display extreme behavior. Due to this, the forecasted conditional covariance matrices will only be estimated using conditional variances estimated from the Real-EGARCH model. The in-sample models will be estimated with conditional variances estimated from both EGARCH and Real-EGARCH models, but the forecasted covariance matrices will only use Real-EGARCH forecasted conditional variances.

ii. DCC-GARCH Models

In this section, I will present the estimated coefficients of the DCC models, including the GFT-DCC variations, and discuss the in-sample and out-of-sample log-likelihood comparisons.

[5] A table with the log-likelihood for each asset individually is available in the appendix (V.6)

Table V.7 displays the estimated coefficients for the DCC models laid out in the model section. Two estimated coefficients are shown for each coefficient of each DCC model, the first being the estimated coefficient from a model using the conditional variances derived from the EGARCH model and the second being the estimated coefficient from a model using the conditional variances derived from the Real-EGARCH model.

Table V.7: DCC Model Coefficient Estimates

	DCC-GARCH		RC-DCC		GFT-DCC		GFT-DCC-2	
	a	***b***	***a***	***b***	***a***	***b***	***a***	***b***
EGARCH	0.0030	0.7645	0.0519	0.7693	0.0641	0.5808	0.0641	0.5808
Real-EGARCH	0.0002	0.7673	0.0656	0.7360	0.0770	0.6847	0.1338	0.2477

The first point of note is that the *b* coefficients in the last three models with EGARCH conditional variance specification are all essentially 0, suggesting that the GFT realized correlation has little impact on forecasting conditional covariance matrices. However, this does not hold when comparing the *b* coefficients in the Real-EGARCH specified GFT models. This gives more credence to using only the Real-EGARCH model in the forecast for conditional variances and covariance matrices.

Also, it is important to note the jump in the magnitude of the *b* coefficient when shrinkage was included and then when the shrinkage estimate was doubled. This suggests that the shrunk realized correlation data may give a better signal than non-shrunk realized correlation, and the optimal shrinkage estimator for this application may not be equal to the optimal shrinkage estimator as specified in the models section.

Table V.8: In-Sample and Out-of-Sample Log-Likelihood of DCC Models

	In-Sample		Out-of-Sample
	EGARCH	**Real-GARCH**	**Real-GARCH**
CCC	-3719.16	-3628.08	-9053.88
DCC-GARCH	-3310.52	-3212.16	-3737.34
RC-DCC	-3292.38	-3189.26	-3649.69
GFT-DCC	-3292.93	-3189.27	-3654.77
GFT-DCC-S	-3299.97	-3194.35	-3673.68

The above table suggests that RC-DCC is the dominant model in both the in-sample and out-of-sample fit. Therefore, when computing the ω for each model according to (4.35), $\mathcal{L}_{j\_low}$ is equivalent to $\mathcal{L}_{RC-DCC}$.

Table V.9: Ratio that Model Induces Minimum Information Loss in Comparison to RC-DCC

	In-Sample		Out-of-Sample
	EGARCH	**Real-GARCH**	**Real-GARCH**
ω_{CCC}	4.4581E-186	2.6419E-191	0
$\omega_{DCC-GARCH}$	1.32185E-08	1.13139E-10	8.57916E-39
ω_{RC-DCC}	1	1	1
$\omega_{GFT-DCC}$	0.576315769	0.995347876	0.006226314
$\omega_{GFT-DCC-2}$	0.000502194	0.006157251	3.816E-11

Table V.9 suggests that RC-DCC is the dominant model in both in and out of sample model. Models other than GFT-DCC have essentially a 0% change that it minimizes information loss from the true data generating model than the RC-DCC model. The in-sample data suggests that GFT-DCC performs similarly to RC-DCC, and the double shrunk GFT-DCC model performs worse than the single shrunk GFT-DCC, suggesting that the James-Stein formulation for the shrinkage estimator is effective and does not require further discretionary shrinkage. This fits the theoretical precedent that RC-DCC will outperform the DCC-GARCH and CCC.

The GFT-DCC models outperformed the DCC-GARCH model under both specifications for conditional variance.[6] Lastly, the CCC model was included simply as a check to make sure the models were seemingly correctly specified. The CCC model is nested inside of a DCC model, and therefore a DCC model should outperform, or at least perform exactly as well, as the CCC model. This holds for the in-sample estimation.

When comparing log-likelihoods of the models in the out-of-sample data, the expected results are similarly met. Both of the GFT models were dominant to the DCC-GARCH but far more inferior to the RC-DCC model than it was in the in-sample estimation. This suggests that the GFT transformed signal, in this instance being realized correlation, acts as a worse signal than the untransformed signal.

Additionally, GFT-DCC performed better than GFT-DCC-S, suggesting bringing extreme values towards a more realistic value decreased the forecasting accuracy. This might be due to the condition of the market in the forecast period. The estimation period was a period of relatively low volatility and high returns, while the forecast period experienced a much larger amount of volatility in returns. Decreasing extreme values in the signal might have been beneficial for the estimation period, but the extreme values in the realized correlation during the forecast period may have actually been realistic and shrinking this data removed valuable information from the market. Removing the extreme values would also increase the time estimated variance require to catch up following a shock.

Overall, the RC-DCC was the dominant model considering the log-likelihood of the model at the estimated parameters and the AIC values. The GFT models outperformed the

[6] A table with ω for each asset with GFT-DCC as the base model is available in the appendix (V.10)

traditional DCC-GARCH model in both the in-sample and out-of-sample periods. However, the GFT models performed significantly worse in the out-of-sample estimation period when compared to the dominant RC-DCC model. This may be due to the nature of the returns during the forecast period.

iii. Mean-Variance Optimization

This section reviews the results of the mean-variance portfolio optimization that took the forecasted covariance matrices as inputs. The weights were re-estimated daily in the forecast period with the goal of minimizing portfolio variance. The model that yields weights for each asset that result in the lowest variance in daily returns over the forecast period is the model considered to give the best, most useful, forecasted variances over the entire period.

Table V.11: Variance of Portfolio Returns over Forecast Period by Model

	Variance
CCC	1.2768E-05
DCC-GARCH	1.2789E-05
RC-DCC	1.2797E-05
GFT-DCC	1.2919E-05
GFT-DCC-S	1.3005E-05

These results differ fairly significantly from the results of the out-of-sample forecast evaluation. Here, the CCC model has the lowest variance of portfolio return during the forecast period, followed by the DCC-GARCH model. The RC-DCC model variance is extremely similar to the DCC-GARCH and CCC models. Lastly, both the GFT-DCC models perform worse than the rest of the models. This may, once again, be due to the characteristics of the market during the forecast period. However, these results reinforce the out-of-sample results in section V.ii, suggesting that the GFT models may not perform as well out-of-sample as in-sample, and the GFT models may not be an improvement on the traditional DCC-GARCH.

It is important to note that these differences in variances are extremely small; these models track each other fairly closely in weights and result in extremely similar returns. Therefore, while this is still a criterion to consider, the log-likelihood and AIC criteria might give a better intuition in determining the dominant model.

VI. Conclusion

In this paper, I have evaluated the validity of including the Generalized Fisher Z-Transformation into a covariance forecasting model, estimating if including the signal given by GFT transformed data in the model outperforms the traditional DCC-GARCH model and a DCC model with realized correlation included as a signal. The theoretical outcome is that both the RC-DCC and GFT-DCC models will outperform the traditional DCC-GARCH model in both in-sample model fit and out-of-sample forecast accuracy. This research also discussed the comparative value of a traditional EGARCH model and an EGARCH model that incorporates realized volatility in its estimation.

A key issue is it impossible to find the true conditional covariance given sample data, so it is difficult to evaluate the forecasted conditional covariance. Log-likelihood was used to estimate the fit of the in-sample model and the accuracy of the forecasted covariance, and AIC criterion was used to show the significance of the dominant model over the others. Additionally, portfolio weights were estimated with the goal of minimum variance of portfolio returns. This allowed for another criteria for forecast accuracy, as the model that produces weights that result in minimum portfolio return variance over the forecast period.

The results were as expected for the EGARCH models. The Real-EGARCH model outperformed the EGARCH model in both in-sample and out-of-sample evaluations, and the EGARCH model resulted in non-stationary results for some assets while the Real-EGARCH did not encounter this issue. Therefore, the DCC models used the conditional variances estimated through the Real-EGARCH for forecasting.

Similarly, the results were largely as expected when considering the in-sample estimation. The RC-DCC, which has empirical precedent to outperform DCC-GARCH, fit the in-sample data the best out of all five models soundly, with the GFT models also outperforming the DCC-GARCH and CCC models. Additionally, as the shrinkage increased in the GFT model, the worse fit of the in-sample data. This suggests that the GFT is a good signal for measuring conditional covariance, improving upon DCC-GARCH and performing similarly to the RC-DCC model.

However, the out-of-sample evaluation delivered a mixed bag of results. Once again, in RC-DCC had the most accurate forecasts according to the log-likelihood criteria, and the AIC criteria suggested that there is a very low chance that it is not the dominant model. Similar to the in-sample data, the GFT-DCC models outperformed the DCC-GARCH model. Under the mean-variance optimization criteria, DCC-GARCH and CCC models performed best, while the GFT models delivered the weights with the highest portfolio variance. This suggests that GFT may be a beneficial addition to in-sample estimation, but it may not be a valuable signal in forecasting.

VVII. Appendix

Table II.3: Correlation of Daily Returns in Forecast Period

	CAT	CVX	DIS	GE	GS	HD	IBM	JNJ	JPM
CAT	1.00								
CVX	0.35	1.00							
DIS	0.47	0.29	1.00						
GE	0.27	0.26	0.21	1.00					
GS	0.49	0.42	0.48	0.25	1.00				
HD	0.44	0.24	0.48	0.16	0.48	1.00			
IBM	0.43	0.26	0.47	0.39	0.46	0.46	1.00		
JNJ	0.32	0.38	0.32	0.18	0.43	0.39	0.39	1.00	
JPM	0.46	0.41	0.45	0.31	0.77	0.51	0.52	0.46	1.00
KO	0.27	0.27	0.32	0.12	0.29	0.39	0.24	0.50	0.29
MCD	0.29	0.15	0.29	0.23	0.28	0.37	0.24	0.51	0.33
MRK	0.38	0.33	0.37	0.22	0.43	0.37	0.39	0.61	0.35
PFE	0.30	0.37	0.39	0.15	0.43	0.42	0.38	0.64	0.40
PG	0.14	0.26	0.13	0.21	0.15	0.15	0.27	0.46	0.20
UNH	0.40	0.35	0.49	0.25	0.43	0.52	0.51	0.53	0.52
UTX	0.65	0.27	0.48	0.36	0.53	0.58	0.50	0.38	0.48
V	0.49	0.23	0.48	0.15	0.44	0.51	0.57	0.31	0.47
WMT	0.27	0.31	0.25	0.19	0.31	0.44	0.24	0.41	0.33
XOM	0.37	0.72	0.37	0.29	0.47	0.24	0.26	0.42	0.45
VX	0.09	0.23	0.27	0.17	0.13	0.25	0.19	0.45	0.20

	KO	MCD	MRK	PFE	PG	UNH	UTX	V	WMT	XOM
KO	1.00									
MCD	0.46	1.00								
MRK	0.37	0.35	1.00							
PFE	0.49	0.30	0.68	1.00						
PG	0.50	0.32	0.33	0.28	1.00					
UNH	0.30	0.37	0.52	0.50	0.26	1.00				
UTX	0.35	0.37	0.42	0.37	0.21	0.42	1.00			
V	0.23	0.23	0.34	0.30	0.08	0.47	0.50	1.00		
WMT	0.40	0.34	0.36	0.35	0.39	0.24	0.29	0.17	1.00	
XOM	0.35	0.20	0.38	0.40	0.27	0.41	0.33	0.29	0.25	1.00
VX	0.48	0.34	0.33	0.37	0.46	0.26	0.19	0.05	0.29	0.20

Table V.2: Robust Standard Errors of Real-EGARCH Coefficients

	ω	β	τ1	τ2	u	ε	φ	δ_1	δ_2
CAT	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
CVX	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
DIS	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GE	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
GS	0.001	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000
HD	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
IBM	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
JNJ	0.018	0.000	0.000	0.000	0.000	0.010	0.000	0.000	0.000
JPM	0.006	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.000
KO	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
MCD	0.048	0.000	0.000	0.000	0.000	0.012	0.000	0.000	0.000
MRK	0.040	0.000	0.000	0.000	0.000	0.004	0.000	0.000	0.000
PFE	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
PG	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
UNH	0.009	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
UTX	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
V	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
WMT	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
XOM	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
VX	0.003	0.000	0.000	0.000	0.000	0.004	0.000	0.000	0.000

Table V.3: Coefficients of EGARCH Model

	ω	**β**	**τ1**	**τ2**
CAT	-0.550	0.942	0.021	0.230
CVX	-0.069	0.994	-0.019	-0.062
DIS	-0.091	0.992	0.005	-0.071
GE	-0.381	0.963	0.033	0.088
GS	-10.669	-0.072	-0.150	0.171
HD	-0.068	0.994	-0.013	-0.056
IBM	-0.003	1.000	-0.003	-0.001
JNJ	-0.453	0.960	0.010	0.034
JPM	-0.097	0.991	0.034	-0.061
KO	-0.006	1.000	0.005	-0.040
MCD	-5.144	0.534	-0.123	0.229
MRK	-0.168	0.985	0.038	-0.086
PFE	-0.483	0.955	-0.054	0.112
PG	-0.102	0.991	0.072	-0.011
UNH	-0.145	0.987	-0.013	-0.086
UTX	-0.026	0.997	-0.084	0.103
V	-1.057	0.901	-0.015	0.242
WMT	-0.661	0.938	-0.002	0.163
XOM	-0.125	0.988	-0.057	0.146
VX	-0.867	0.920	0.046	-0.244

Table V.4 Robust Standard Error of EGARCH Coefficients

	ω	**β**	**τ1**	**τ2**
CAT	0.149	0.015	0.002	0.002
CVX	0.000	0.000	0.000	0.000
DIS	0.000	0.005	0.008	0.000
GE	0.056	0.002	0.002	0.001
GS	4.587	0.016	0.009	0.048
HD	0.000	0.000	0.000	0.000
IBM	0.000	0.000	0.000	0.000
JNJ	1.362	0.061	0.041	0.011
JPM	0.000	0.000	0.000	0.000
KO	0.000	0.000	0.000	0.000
MCD	9.272	0.012	0.006	0.077
MRK	0.000	0.000	0.000	0.000
PFE	0.429	0.023	0.001	0.004
PG	0.001	0.002	0.001	0.000
UNH	0.000	0.001	0.000	0.000
UTX	0.027	0.002	0.001	0.000
V	4.254	0.101	0.004	0.038
WMT	0.157	0.008	0.000	0.001
XOM	0.015	0.006	0.003	0.000
VX	0.000	0.009	0.001	0.000

Table V.6: Log-Likelihoods for EGARCH and Real-GARCH

	In-Sample		Out-of-Sample	
	EGARCH	**Real-EGARCH**	**EGARCH**	**Real-EGARCH**
CAT	1519.39	1622.33	555.23	522.43
CVX	1719.27	1770.95	-12529.34	644.24
DIS	1797.83	1902.55	-4130.03	653.40
GE	1720.61	1872.14	603.35	606.36
GS	1609.75	1767.04	695.39	655.91
HD	1800.26	1891.25	-10539.67	688.16
IBM	1803.93	1907.16	461.13	694.65
JNJ	1911.76	2049.61	715.50	559.09
JPM	1711.60	1799.66	-4361.19	679.56
KO	1926.66	2030.14	-10615.01	774.41
MCD	1862.91	2029.29	698.39	552.00
MRK	1759.89	1869.51	-11797.57	675.86
PFE	1776.13	1862.46	713.54	692.88
PG	1877.42	2002.96	-45.24	712.41
UNH	1730.12	1798.92	-4578.17	704.16
UTX	1770.10	1893.09	710.95	704.47
V	1781.68	1894.31	725.60	706.00
WMT	1783.31	1921.28	730.70	683.49
XOM	1785.60	1881.28	738.00	700.28
VX	1778.97	1900.41	-10669.74	682.80

Table V.10: Ratio that Model Induces Minimum Information Loss in Comparison to GFT-DCC

	In-Sample		Out-of-Sample
	EGARCH	**Real-GARCH**	**Real-GARCH**
ω_{CCC}	7.7356E-186	2.6543E-191	0
$\omega_{DCC-GARCH}$	2.29363E-08	1.13668E-10	1.37789E-36
ω_{RC-DCC}	1.735159879	1.004673868	160.6086674
$\omega_{GFT-DCC}$	1	1	1
$\omega_{GFT-DCC-2}$	0.000871387	0.006186029	6.12883E-09

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